Global Journal of Enterprise Information System

DOI: 10.18311/gjeis/2016/7288

A Clarke and Wright Improved Algorithm to Solve the Vehicle Routing and Traveling Salesman Problem

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Abstract

Vehicle Routing Problem (VRP) and Traveling Salesman Problem (TSP) are well known transportation problems. The problems can be seen in all the industries that involves goods distribution and transportation scheduling. Finding the shortest distance with respect to the given constraint contribute highly to save money and energy consumption. This paper investigates the possibility of creating a cellular application that can provide an instant routing plan through a simple heuristic (Clarke and Wright) in order to avoid the usage of more complicated approaches as metaheuristics and exact methods that normally taking very long CPU time.

Keywords: Clarke and Wright, Domain Reduction, Traveling Salesman Problem

(Article chronicle: Acceptance: January-2016; Published: March-2016)

1. Introduction

Increasing demands on transportation forced companies to try to optimize their costs. The Traveling Salesman Problem (TSP) is considered to be a combinatorial problem. The objectives of TSP are to find an Optimal Hamiltonian Circuit (OHC) to complete the task. TSP problem has been proven to be NP-complete. ZHANG et al. used multi-objective ant colony optimization to tackle TSP[2]. Kuchaki Rafsanjani et al. used an improved genetic algorithm and local search hybridization to solve the problem[3]. Kara et al. proposed a new integer linear programming formulation for the traveling salesman problem[4].

The Vehicle Routing Problem (VRP) is an important problem in the distribution network and has a significant role in cost reduction and service improvement. The problem is one of visiting a set of customers using a fleet of vehicles, respecting constraints on the vehicles, customers, drivers etc[5]. The main objective is to minimize the cost for the routing of each vehicle. The problem of vehicle scheduling was first formulated in 1959[6] and may be stated as a set of customers, each with a known location and a known requirement for some commodity that is to be supplied from a single depot by delivery vehicles, subject to the following conditions and constraints:

- The demands of all customers must be met.
- Each customer is served by only one vehicle.
- The capacity of the vehicles may not be violated (for each route the total demands must not exceed the capacity).

The objective of a solution may be stated, in general terms, as that of minimizing the total cost of delivery, namely the costs associated with the fleet size and the cost of completing the delivery routes[3]. The problem frequently arises in many diverse physical distribution situations. For example bus routing, preventive maintenance inspection tours, salesmen routing and the delivery of any commodity such as mail, food.

The Vehicle Routing Problem (VRP), and the Travelling Salesman Problem (TSP), are important problems in the field which many fields. These algorithms play a significant role in reducing the cost and improving the service. They are well-known problems and involve many mathematical concepts. This study will investigate the TSP, which deals with the following situation: If a number of cities (or customers) is given as well as the distance between each pair of these cities. Then the question would be what the shortest possible route is between origin and destination, given that each destination is visited only once.

The paper provides a hybrid approach that combined Clarke and Wright algorithm with domain reduction to solve vehicle routing problem and travelling salesman problem. VRP and TSP are two well-known important problems. Minimizing the distances using the hybrid approach as a cellular application will contribute highly to:

- Minimizing petrol consumption.
- Minimizing the pollution through lowering gas emissions.
- Reducing traffic.

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2. Mathematical Formulation

The CVRP is to satisfy the demand of a set of customers using a fleet of vehicles with minimum cost. Achuthan et al7 described the problem as follows:
Let

- C = {1, 2,…, n}: The set of customer location.
- 0: Depot location.
- G = (N,E): The graph representing the vehicle routing network with N = {0,1,…,n} and E = {(i,j):i,j∈N, i<j}.
- q_j: Demand of customer j.
- Q: Common vehicle capacity.
- m: Number of delivery vehicles.
- c_{ij}: Distance or associated cost between locations i and j.
- L: Maximum distance a vehicle can travel.
- P_j: A lower bound on the cost of traveling from the depot to customer j.
- l (S): Lower bound on the number of vehicles required to visit all locations of S in an optimal solution. Note that S ⊆ C and l (S) ≥ 1.
- $\bar{S}$: The complement of S in C.
- x_{ij}: 1,2, or 0.

The problem is to:

$$\text{Minimize } Z = \sum_{i\in N}\sum_{i<j} c_{ij}x_{ij} \quad i \in N, i<j \tag{1.1.1}$$

Subject to

$$\sum_{i\in C} x_{0i} = 2m, \quad i \in C \tag{1.1.2}$$

$$\sum_{j<i} x_{ij} + \sum_{i<j} x_{ji} = 2, \; i \in C \tag{1.1.3}$$

$$\sum x_{ij} \le |S| - \ell(S), \; i,j \in S, \;\; S \subseteq C, 3 \le |S| \le n\text{-}2 \tag{1.1.4}$$

$$X_{ij} = 1,2, \text{ or } 0$$

In addition, this paper considers the following mathematical formulation for the travelling salesman problem[8].

For $i = 0, ..., n$, let u_i be an artificial variable, and finally take c_{ij} to be the distance from city i to city j.

$$\text{Min} \sum_{i=0}^{n} \sum_{j\ne i, j=0}^{n} c_{ij}x_{ij} \tag{1}$$

$$u_i \in \mathrm{Z} \quad i, j = 0,...,n \tag{2}$$

$$\sum_{i=0, i\ne j}^{n} x_{ij} = 1 \quad i, j = 0,...,n \tag{3}$$

$$\sum_{j=0, j\ne i}^{n} x_{ij} = 1 \quad i, j = 0,...,n \tag{4}$$

$$u_i - u_j + nx_{ij} \le n-1 \quad 1 \le i \ne j \le n \tag{5}$$

$$x_{ij} = \begin{cases} 1 \\ 0 \end{cases} \tag{6}$$

Where (1) is the objective function. (2) Implies that u_i integers. (3) Is to get sure that each city visited at most once. (4) Serves the fact that all cities must be visited. (5) Is to get sure that there is only one tour for the problem. (6) Implies that the value of x is 1 if the path goes from city i to city j and 0 otherwise.

3. Constraint Programming

Constraint Programming techniques have been developed since about early 1990s. They have two common features constraint propagation and distribution (labeling) connected with search.

Constraint propagation will lead to Constraint Programming, CP. It would automatically remove from the domain of variables all values that do not fulfill constraints. Let us consider these examples:

- Let $X\text{-}Y = 3$ or $X<Y$. These 2 given constraints would provide information about the values of the variables X and Y, but it is in a poorly usable form. CP will work to simplify such information. If we have beside $X\text{-}Y = 3$ the other information $X+Y = 7$, then the solution would be: $X = 5$ and $Y = 2$. This simplification will be carried out by a special algorithm, the constraint solver, a fixed part of the CP.
- If we have two variables x and y, with x ∈ {1..5} and y ∈ {1..6}. We introduce here a constraint with x>y+1, then the constraint propagation will reduce the domains to the following values: x ∈ {3, 4, 5} and y ∈ {1, 2, 3} because values {1, 2} from x domain do not fulfill the constraint x>y+1 and the y values {4, 5, 6} also conflict with the given constraint.
- In the last example, if we add another constraint x+y = 6, then none of the values can be removed.

Usually we don't have the joy of such simple constraints. They are often connected with each other. Therefore, constraint propagation would not remove all values that are in conflict with all constraints and its performance is measured as a trade-off between number of removed values and execution time.

Actually, constraint propagation does not lead to the solution (example above). This explains the need for always to add to constraint propagation a distribution connected with search. Distribution is based on incorporation of an additional constraint, often it is a constraint about equality of one variable to one value. A major task of the distribution is to find or choose a proper variable and a suitable value. As soon as this is achieved, a consistency is checked and there will be three possibilities:

(i) A solution is found,
(ii) Variables domains are narrowed, but there is no unique solution, so distribution is made with another variable,

(iii) The additional constraint is inconsistent with other constraints, so the backtrack is made and from chosen variable domain a chosen value is removed.

This is an iterative process and it is called *search*. Search is responsible for stopping after finding first solution or some number of solutions or all solutions. Search forms a search tree, where each node is a state of variables.

4. Computations

Improved Clarke and Wright algorithm when applied to solve TSP and VRP well known benchmark problems are shown in the Figure 1 & 2.

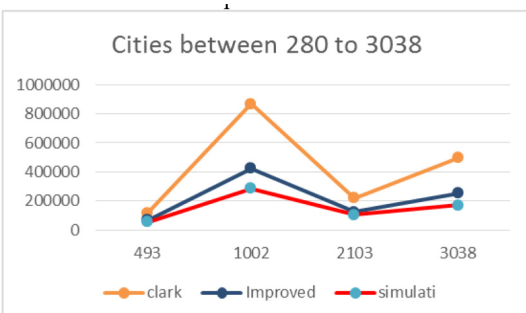

Figure 1. TSP Improved Clarke and Wright.

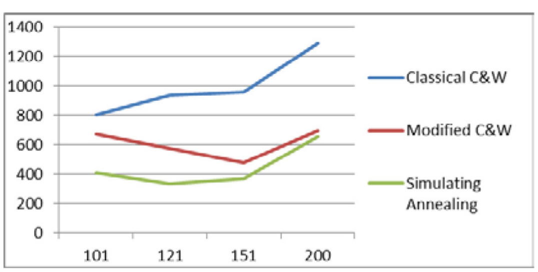

Figure 2. VRP Improved Clarke and Wright.

5. Conclusion

Domain reduction is very effective when combined with Clarke and Wright saving algorithm. For large instance the improvement of the traditional Clarke and wright reached 48-50 % (as suggested by Figure 1 and 2). Improving a simple algorithm like Clarke and wright is very important if a cellular application was developed to optimize the routes for maintenance workers, salesmen and distribution vehicles and networks. The improved Clarke and Wright is fast, easy to implement and understand as well as require a normal (non-advanced) computer to produce an accurate results that are not far from the results obtained by more complicated meta-heuristics such as simulating annealing. The usage of the cellular application will contribute highly to energy savings and reducing the pollution. The hybrid approach of using domain reduction to improve the results should be considered to optimize the shortest path problem in any future work.

6. References

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Citation:

Mamoon Alameen, Rasha Aljamal and Sadeq Damrah
"A Clarke and Wright Improved Algorithm to Solve the Vehicle Routing and Traveling Salesman Problem"
Global Journal of Enterprise Information System, Volume 8 | Issue 1 | January-March 2016 | www.informaticsjournals.com/index.php/gjeis

Conflict of Interest:

Author of a paper had no conflict neither financially nor academically.