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Two Player Decision Behaviors Changing in Repeated Game

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ABSTRACT

This paper studies Stackelberg model in repeated game with the learning attitude of the follower and leader-follower. Leader player (firm-A) and follower player (firm-B) produce the homogeneous good in initial period. Follower does not want to be a follower always, but want to work with equal profit gainer at least. In finite periods play, both firms tent to produce the homogeneous good as per Cournot Game due to having leader-firm's farsightedness in production of good. Both the Firms collude and produce less than Nash-Cournot equilibrium to maximize its profit in each period.

KEYWORDS

Stackelberg game	Cournot game
SPE	Repeated game

INTRODUCTION

Game theory is a mathematical tool through which game situations determine a final outcome to the conflict. Each one participating like a player can control the situation partially, but any player has complete control. Each player has certain personal preferences about the possible outcome of the game and he makes an effort to obtain the most beneficial outcome for him, but he knows that the other ones make the same thing (this is rationality). It is well known that it is more advantageous for a firm to be a leader than a follower in Stackelberg duopoly without product differentiation. Stackelberg's book "Marktform und Gleichgewicht" (1934) proposes a sequential model of market economy including one leader and one follower. The leader moves first and makes her decision taking into consideration the reaction of the follower. The leader knows the demand function and her rival reaction function. The follower also knows the demand function and can set his own output level according to any possible function of the quantity set by the leader, with the expectation that the leader will not counter-react. Similarly, the leader may expect the follower to conform to the choices given by his reaction function. At Stackelberg equilibrium, both firms optimize given their beliefs and the frims' beliefs are self-fulfilled for these equilibrium choices (Tirole 1988, Vives 1999).

The standard Stackelberg oligopoly equilibrium model may be conceived as a subgame perfect Nash equilibrium of a two stage game, where each player moves in a prescribed order (Fudenberg and Tirole 1991; Osborne and Rubinstein 1994). One salient feature of Stackelberg duopoly model when frims compete on quantity is the following: under both assumptions of linear market demand and constant identical marginal costs, the leader always achieves a higher payment than the follower. However, tackled in a T-stage game, with one firm per stage, the Stackelberg model may generate a situation in which the Cournot profit may exceed the leader's profit (Anderson and Engers 1992). In this model the Stackelberg price becomes arbitrarily small relative to the Cournot price as the number of firms (stages) becomes large, and this effect dominates the large output of the first leader.

Gal-Or (1985) shows that a leader obtains relatively higher profits when the slope of firms' reaction functions is negative, while Dowrick (1986) shows the opposite when these slopes are positive. We rather focus on the conditions on the slope of the followers' reaction functions as rationally expected by the leaders. It enables to circumvent the conditions under which leaders may achieve better payments than followers. Daughety (1990) considers a parameterized class of Stackelberg markets and shows that all sequential-move structures are beneficial compared to the simultaneous-move Cournot markets.

The objective of this paper is threefold. First, we see the reaction of the leader and follower firm to produce the output of homogeneous good in initial stage, subject to some plausible market assumptions. Second, we study the learning behavior of the follower and leader-follower in t-stage (learning curve (Yelle, 1979)) supply a homogeneous product in Cournot game with its rival firm in a noncooperative manner. Third, Leader's farsightedness and regular learning of the follower-firm maximize the profit of both in each stage of the game. Farsightedness of the firms makes them collude to maximize upto infinite periods.

Our results are supported by the Example: the level of output increases, Stackelberg markets yield higher output, higher consumer rents and higher welfare levels than Cournot markets. We find considerable deviations from the subgame perfect equilibrium prediction in Stackelberg markets to Cournot output.

The remainder of the paper is organized as follows: Section 1 introduction the Stackelberg model and its different results, interrelationship of the Cournot game and sequential move game Stackelberg. In this Section 2, we introduce notations, assumptions, definitions, preliminary concepts. Section 3 provides the model of the game and graphical presentation of results. Section 4 provides the examples which illustrate the deviation of the follower to use the learning strategy with respect to the maximizing profit of the leader. Section 5 presents the conclusion of the paper.

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PRELIMINARIES OF TWO PLAYERS GAME

Let Γ=$(Q_1, Q_2 : \pi_1, \pi_2)$ be 2-player game, where $i = (1,2)$ is the set of players.Q_i is the set of actions of player i and $\pi_i: Q_1 \times Q_2 \rightarrow \mathbb{R}$ is player i 's payoff function.

The associated infinitely repeated game with discounting is denoted by $\Gamma^{\infty}(\delta)$ $where\ \delta \in (0,1)$ is the discounted factor. If $Q(t) = (Q_1(t), Q_2(t))$ is the vector of action played in period t, then $\{Q(t), \dots Q(t)\}$ is a history h of length. A pure strategy σ_i of player i in $\Gamma^{\infty}(\delta)$ is a sequence of function σ_i^t or $\sigma_i(t)$ from the set of all histories of length $(t-1)$ to Q_i so $\sigma_i^1 \in Q_i$ is the initial action of player i.

A stream of action profile $\{Q(t)\}_{t=1}^{\infty}$ is reffered to as an outcome path and is denoted by S any strategy profile$\sigma = (\sigma_1, \sigma_2)$ generates an outcome path $S(\sigma) = \{Q(t)\}_{t=1}^{\infty}$ defined inductively by $Q(\sigma)(1) = \sigma^1 . Q(\sigma)(t) = \sigma^t\big(Q(\sigma)(1), \dots, Q(\sigma)(t-1)\big)\ if\ \ t > 1.$

The value $\pi_i(Q(t)$ denotes the payoff of player i in period t when the outcome in this period is $Q(t)$ and $\pi_i^{\delta}(S)$ denotes the averages discounted payoff of player i for the outcome path $S = \{Q(t)\}_{t=1}^{\infty}$:$\pi_i^{\delta}(S) = (1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}\pi_i(q(t))$ then ,the averages discounted payoff of player i in $\Gamma^{\infty}(\delta)$ obtained with the strategy profile $\sigma = (\sigma_1, \sigma_2)$ is $\pi_i^{\delta}(\sigma) = \pi_i^{\delta}(S(\sigma))$.

A strategy profile $\sigma = (\sigma_1, \sigma_2)$ is a Nash Equilibrium in $\Gamma^{\infty}(\delta)$ if for $i = (1,2)$, σ_1 is a best response σ_2. And it is a Subgame Perfect Equilibrium (SPE) in $\Gamma^{\infty}(\delta)$ if after every history$h$, σ_h(i.e. the continuation of σ after h) is a Nash equilibrium in the corresponding subgame.

Assumptions: In the specific game we consider the Cournot Model with perfect monitoring. two firms produce a homogeneous good at cost function $C(Q) > 0$. The industry inverse demand function is denoted by $P(Q)$ and payoff function is denoted by $\pi_i(q_1, q_2) = \big(P(.)\big)q_i - C(.)q_i$, where q_i is the output of the firm i.

A.1: Inverse Demand Function $P(.): \Re_+ \rightarrow \Re_+$ is continuous, differential and with $P'(Q) < 0\ for\ \forall\ Q > 0$ such that $P(Q) > 0, \lim_{q\rightarrow\infty} P(Q) = 0$ and $P(0) > c.$ Let

$Q_1^D(Q_2)$ be a single period best response to Q_2 that is $Q_1^D(Q_2)$ satisfy $\pi_1(Q_1, Q_2^D) \geq \pi_1(Q_1, Q_2)$ for $\forall\, q_1 \in Q_1$.

A.2: $Q_1^D(Q_2)$is well defined, unique and$Q^D(q) = q_1^D(q_2)$, where $q = q_2$ is a continuous, non-increasing function. Let $\pi_i^D(q)$be player $i's$ best response payoff when other player play according to q_2, that is $\pi_i^D(q) = \pi_i(q_1, q_i^D(q_2))$.

A3: Learning function of the firm is $v_t = v_1 t^{\log\phi/\log 2}, where\ 0 \leq \phi \leq 1$ is firm's learning parameter and t is period of the game. Learning aspect generates the farsightedness and come together to produce meet the demand.

MODEL OF THE GAME

In this section we review the basic results of Both Cournot and Stackelberg game in a static linear demand at Initial Period, $i = 1, 2$ number of player (1-Leader and 1-Follower).Marginal cost of firm $i = c_i(t)$ Market demand $p(t) = a - bq(t)$,where *q* is the quantity, and *a and b* parameters. In Cournot Competition, the firms play a quantity-setting game with simultaneous moves. In case of Stackelberg Competition, the firms play also a quantity setting game with sequential move, Stackelberg –Leader move first, Follower-firm play its best response then they produce the quantity non-negative amounts. In Cournot Model, Equilibrium

$q_2^*(t) = (a + c_1 - 2c_2)/3b\,(t)$,

$q_1^*(t) = (a + c_2 - 2c_1)/3b\,(t)$ and Profit function

$\pi_1(t) = (a + c_2 - c_1)^2/9b(t)$

, $\pi_2(t) = (a + c_1 - c_2)^2/9b\,(t)$

and price of the quantity $p(t) = (a + c_1 + c_2)/3\,(t)$.

and In Stackelberg model, Equilibrium,$q_L^*(t) = (a + c_2 - 2c_1)/2b(t)$, $q_F^*(t) = (a + c_1 - 2c_2)/2b(t)$, and profit function $\pi_L(t) = (a + c_2 - 2c_1)^2/8b\,(t)$, $\pi_F(t) = (a + 2c_1 - c_2)^2/16b\ (t)$ and price of the quantity $p(t) = (a + 2c_1 + c_2)/4\,(t)$.

Learning curve of the firm [Yelle, 1979]: $v_t =$ $v1t\log\phi\log 2, where\ 0 \leq \phi \leq 1$

Leader produces q_L^* in S-game in $t = 1$ period and q_L in $t = T - 1$ period.

$q_L(t = T - 1) \in (q^*, q_L^*)$, $q^* < q_L < q_L^*$, and $\pi_L(q^*, q^*)(t = T) < \pi_L\big(q_L, B(q_L)\big)(T - 1) <$ $\pi_L(q_L^*, q_F^*)(t = 1)$, Follower produces q_F^* in S-game in $t = 1$ period and q_F in $t = T - 1$ period. $q_F(t = T - 1) \in (q_F^*, q^*)$.

$q_F^* < q_F < q^*$ and $\pi_F(q^*, q^*)(t = T) > \pi_F\big(q_L, q_F =$ $BqLT-1>\pi FqL*,qF*(t=1)$

Leader-firm have a farsightedness aspect due to Follower-firm's learning, Leader offer its agreement to the follower to produce more quantity than one period. If this agreement increases in total profit K then follower-firm has to share this Extra profit (Fig: 1) as per agreeing offered conditions by the leader-firm ($x\ is\ an\ arbitrary$). To get this profit leader firm agree to share in cost of extra units by follower-firm

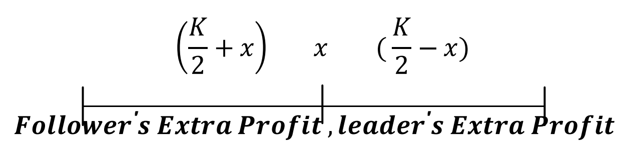


Fig: 1 (Extra Profit sharing in between Stackelberg leader and Stackelberg follower-firm)

After Agreement: Both firms increase its profit in $t = T - 1$ periods and make share of it.

Leader-firm's profit be higher after adding gain from agreement with the follower-firm-

$$\pi_L(q_L, q_F) + \left(\frac{K}{2} - x\right) \geq \pi_L(q_L^*, q_F^*)$$

Follower-firm's profit be higher after adding gain from agreement with the leader-firm-

$$\pi_F(q_L, q_F) = \left(\frac{K}{2} + x\right) + \pi_F(q_L^*, q_F^*)$$

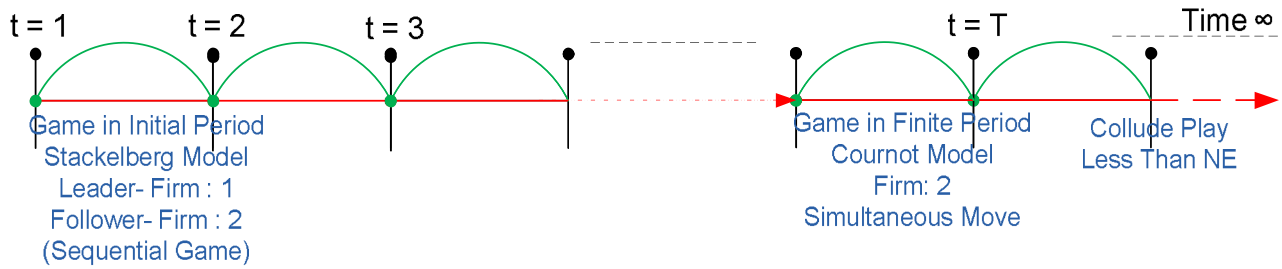
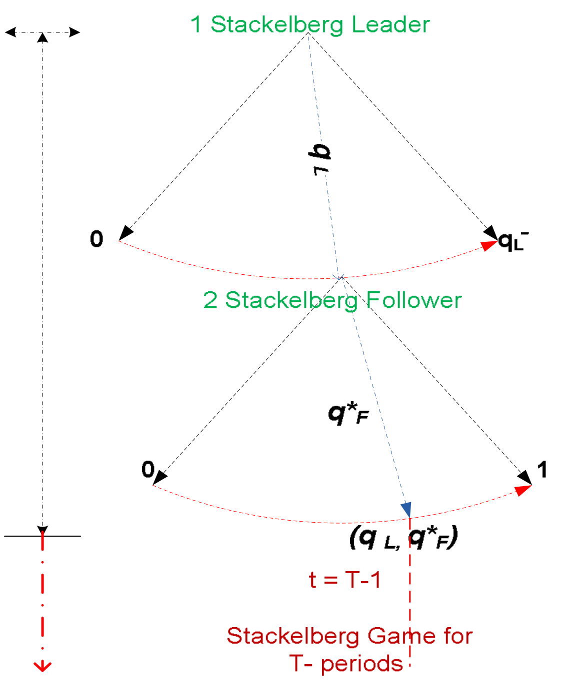


Fig: 2 (Model of the Game, upto $t = T - 1$ periods S-game for $t = T$ period C-game and firms' collusion for infinity to maximize profit**)**



i) (S-game for $t = T - 1$ periods)

Farsightedness of the leader and observation on the follower's regular learning, convert game into Cournot-game and both inclined to collude for the maximization of its profit. Produce less than N-Cournot equilibrium which maximizes its profit to play upto infinity (Fig: 2).

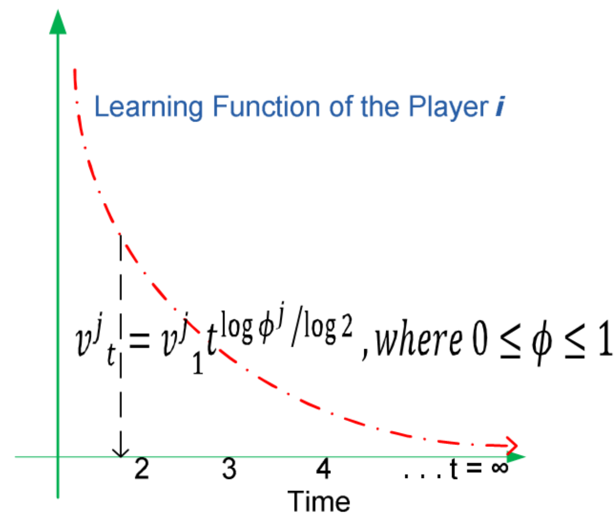


Fig: 3- ii) Firm's learning curve in each period

Following fig: 4 i) follower-firm achieve the leader's learning in period $(t = T)$, just before it both firm agree to play as per S-game and in this period firms agree to play simultaneous game (C-game). When both leader and follower have dynamic regular learning play simultaneous game in periods $(t = T - 1$ (fig: 4 ii).

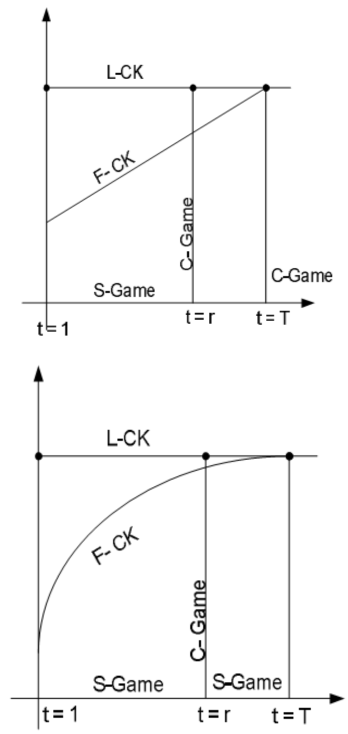


Fig: 4 i) leader and follower's learning

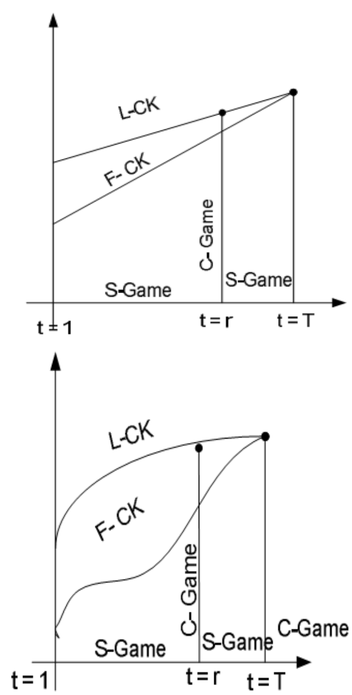


Fig: 4 ii) leader and follower's learning

Example: Player: Firm 1, Firm 2, linear inverse demand function: $P(Q = q_1 + q_2)(t) = (30 - Q)(t)$ Linear cost function: $C_i(q_i)(t) = 6q_i(t), i = 1,2.$

In Cournot market: $\pi_1\ (q_1\ , q_2)(t) = q_1(P(Q) -$ *Ct=q130–6–q1–q2(t), if Q≤30*, To find out firm 1's best response to any given output q_2 of firm 2.we need to study firm 1's profit as a function of its output q_1 for given $q_2 = q_2^*.\pi_1^{'}(q_1, q_2^*) = \frac{\partial}{\partial q_1}\{q_1(24 -$ *q1–q2*=0*, then *q1at given q2* = 12 (24–q2*)*, due to similarity of $q_2 = \frac{1}{2}\ (24 - q_1^*)$, putting the best responses, we get Nash equilibrium $(q_1^*,\ q_2^*)(t)$ =8,Total produced quantity $Q = 2 \times q_1^* = 16, P(Q) = 30 - 16 = 14, \pi_i^t = 8 \times 8 = 64$,cunsumer surplus =128 and total welfare=256.

In Stackelberg market: Firms {leader (L), follower (F)} choose their quantities sequentially. Stackelberg leader (L) decides on its quantities q_1^L ,it is a game of complete information, knowing q_1^L - Stackelberg follower (F) decides on its quantity q_2^F. Leader firm's strategy to produce = q_1,

Follower firm's strategy to play his best response at given $q_1, q_2 = \frac{1}{2}\ (24 - q_1^*)(t)$, now first order condition at profit function $\pi_1^{'}(q_1, q_2)(t) = \frac{\partial}{\partial q_1}\{\frac{1}{2} q_1(24 - q_1)(t) = 0\ \Rightarrow$

$q_1(t) = 12, and\ q_2(t) = 6$, total produced quantity $Q = q_1 + q_2 = 12 + 6 = 18$, profits of the Stackelberg Leader firm $\pi_1^L(t) = 72$, profit of the Stackelberg follower firm ,$\pi_2^F(t) = 36$, consumer surplus = 162, and total welfare = 270.

Quantity comparison of the first player (Stackelberg leader firm and Cournot firm)-

$q_{1_{SL}}^*\ (t) >\ q_{1_{CF}}^*(t)$ (i)

Quantity comparison of second player (Stackelberg follower firm and Cournot firm)-

$q^*_{2_{SF}}(t) < q^*_{2_{CF}}(t)$ (ii)

Total Quantity comparison-

$(Q_{SE} = q^*_{1_{SL}} + q^*_{2_{SF}})(t) > (Q_{CE} = q^*_{1_{CF}} + q^*_{2_{CF}})(t)$ (iii)

Payoff comparison of the first player (Stackelberg leader firm and Cournot firm)-

$\pi_{SL}(q^*_{1_{SL}}, q^*_{2_{SF}})(t) > \pi_C(q^*_{1_{CF}}, q^*_{2_{CF}})(t)$ (a)

Payoff comparison of the Second player (Stackelberg follower firm and Cournot firm)-

$\pi_{SF}(q^*_{1_{SL}}, q^*_{2_{SF}})(t) > \pi_C(q^*_{1_{CF}}, q^*_{2_{CF}})(t)$ (b)

Total Payoff comparison in Stackelberg and Cournot game-

$\pi(q^*_{1_{SL}}, q^*_{2_{SF}})(t) > \pi(q^*_{1_{CF}}, q^*_{2_{CF}})(t)$ (c)

Total welfare comparison in Stackelberg and Cournot game-

$TW_{SE}(t) > TW_{CE}(t$ (d)

Simple to observe the result of the firms in t-period, Firms are the firms which produce the homogeneous good for the welfare of the firms in long run aspect, at least Stackelberg leader have farsightedness to produce the good which inclined to welfare of the firm only to compete the market demand. in initial period, firms play Stackelberg game, in which leader firm are inclined to maximize its profit but unable to fulfill market demand, declaring its quantity to produce -knowing the quantity decided by the leader firm, follower firm choose its best response quantity which maximize its profit in t-period.

Above results shows: Total profit in S-game in initial stage is less than the profit in C-game on which firms inclined to make it own in $(t = T)$. Learning and farsightedness of the firms give this opportunity to maximize the profit without loss of generality. Further collusion of the firms increases the profit with long run play. Joint-profit maximization implies, $q_{i=1}(t) = 6, Q = q_1 + q_2 = 12, \ \pi_{i=1}(t) = 72$. Long run aspect of the firms makes it possible to maximize its profit with the agreement process.

CONCLUSION

In this paper, we have drawn the firms' aspect to maximize its profit without worrying about the total social welfare reduced in each period. Firms' long run play makes heavy loss in social welfare of the society repeatedly. Firm play with co-operation only with competitive the firm which can leave behind in future rationally. Here it is a difficult to define which firm is leader and which one is the follower. As in studied paper, Leader-follower firm are distinct due to having some basic economic ability to produce a wide range of outputs with reasonable profit margin and their size. We can distinct the leader and follower by emerging in industries comprised of some well established firms with sound assets, and other newer, more fragile firm. The follower firms, being less resilient to business shocks, may hence adopt a follower role in the market, awaiting for the more established leader firms to stabilize before making decisions on their own production levels. Undoubtedly, obtained equilibrium solution is a fixed point of the dynamic process in which the leader-and follower-firms readjust output levels according to the strategic market assumptions. Farsightedness of the leader works properly to make the proposal of agreement to reduce the production and maximize the gain from social welfare or from the consumers' surplus in long run.

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