Global Journal of Enterprise Information System

DOI: 10.15595/gjeis/2014/v6i2/51841

A Combinatorial Optimization Approach to Solve the Synchronous Optical Network (SONET) Problem

Mamoon Alameen[1*], Mohammad Abdul-Niby[2], Tarek Selmi[2] and Sadeq Damrah[1]

[1]*Department of Mathematics, The Australian College of Kuwait; m.radiy@ack.edu.kw, s.damrah@ack.edu.kw*

[2]*Department of Electronic Engineering, The Australian College of Kuwait; m.nibi@ack.edu.kw, t.selmi@ack.edu.kw*

Abstract

Synchronous Optical Network (SONET) is a network with a fast service capability. Most of businesses that deal with finance are willing to pay high rates of money just to get reliable continuous connections. The problem is one of assigning each customer to a ring without violating the capacity constraints. The goal is to minimize the size of each ring without sacrificing each customer demands. This paper is to introduce a new effective mathematical approach to optimize the SONET network. The main idea is to transform SONET problem formulation to the capacitated vehicle routing problem.

Keywords: Branch and Cut, Capacitated Vehicle Routing Problem, Clarke and Wright, SONET

1. Introduction

Synchronous Optical Network (SONET) is known to be fast and self healing network. SONET equipment can detect the problem in less than a millisecond and react quickly to solve it then resume communication. Some industries like banks, broker houses and credit cards, prefer to pay more money just to insure safe and reliable communication network. The high level of service availability offered by SONET justifies the growing demands to use the services. In the other hand, one of the crucial parts in setting SONET is optimizing SONET rings.

The SONET ring consists of a certain number of nodes. A demand between some pair of nodes (not all pairs have demands) is given in unites of DS3 (51.84Mbits/sec). The demand is an estimate of the number of circuits needed to provide communications between that pair of nodes. The problem is to find a minimum cost SONET ring network (given nodes, links and demands) such that resulting equipment and fiber links have sufficient capacity to satisfy the demands.

The problem of Optimizing SONET ring can be seen as mix integer linear programming. Furthermore the simple form of optimizing SONET ring problem can be considered as same as (the well known) Capacitated Vehicle Routing Problem CVRP.

The Capacitated Vehicle Routing Problem CVRP was first formulated by Dantzig and Ramser[1] and may be stated as a set of customers, each with a known location and a known requirement for some commodity. The customers are to be supplied from a single depot by delivery vehicles, subject to the following conditions and constraints:

(a) The demands of all customers must be met.
(b) Each customer is served by only one vehicle.
(c) The capacity of the vehicles may not be violated.

Hence, each node in SONET ring can be treated as a customer in CVRP. The capacity of the links in SONET is as same as the capacity of the vehicle in CVRP. In general, optimizing the SONET ring problem can be considered as one route CVRP.

2. CVRP Mathematical Formulation

Achuthan et al.[2] described the problem as follows:

Let

- $C = \{1, 2, \ldots, n\}$: the set of customer location.
- 0: depot location.
- $G = (N,E)$: the graph representing the vehicle routing network with $N = \{0,1,\ldots,n\}$ and $E = \{(i,j): i,j \in N, i<j\}$.
- q_j : demand of customer j.
- Q: common vehicle capacity.
- m: number of delivery vehicles.
- c_{ij}: distance or associated cost between locations i and j.
- L: maximum distance a vehicle can travel.
- P_j: a lower bound on the cost of traveling from the depot to customer j.
- $\ell(S)$: lower bound on the number of vehicles required to visit all locations of S in an optimal solution. Note that $S \subseteq C$ and $\ell(S) \le 1$.

* *Author for correspondence*

- S: the complement of S in C
- x_{ij}: 1,2, or 0

The problem is to:

$$\text{minimize} Z=\sum_{i\in N}\sum_{i<j} c_{ij}x_{ij} i\in N, i<j \quad (1.2.1)$$

subject to

Constraints (1.2.2) and (1.2.3) known as degree constraints.

$$\sum_{i\in C} x_{0i}=2m, i\in C \quad (1.2.2)$$

$$\sum_{j<i} x_{ij}+\sum_{i<j} x_{ji}=2, i\in C \quad (1.2.3)$$

$$\sum x_{ij}\le |S|-\ell(S),\ \ i,j\in S,\ \ S\subseteq C, 3\le |S|\le n-2 \quad (1.2.4)$$

$$x_{ij}=1, 2, \text{or } 0 \quad (1.2.5)$$

Constraint (1.2.2) specifies that the number of vehicles leaving and returning to the depot are m. Constraint (1.2.3) specifies that each customer is visited by only one vehicle. Constraint (1.2.4) is referred to as subtour elimination constraints, which prevent subtours from forming loops disconnected from the depot, or eliminate tours that connected to the depot but violate the capacity restriction. Note that a connected component of a weighted or un-weighted graph defined over the set of customers is called a subtour. The subtour will be called a tour if it's connected to the depot in a graph defined over all locations. Constraint (1.2.5) specifies that if a vehicle travel on single trip between i and j then the value of x_{ij} will be 1,and if i=0 and (0,j,0) is a route then the value of x_{ij} will be 2, otherwise the value of x_{ij} will be 0.

3. The Advantages of Optimizing SONET Ring using CVRP Techniques

Observing the work of Bernardino et al.[3], Kim et al.[4], Karunanithi and Carpenter[5], and many other popular publications, the following points can be raised:

- No exact method applied to optimize SONET ring.
- The suggested approaches used to solve small size problems (6–20 nodes).

Hence, considering a SONET ring as one-route CVRP can provide the followings:

- Applying exact methods like branch and bound branch and cut, etc… to get the exact solution.
- The ability to solve instances with hundreds of nodes.
- Apply more conditions to increase efficiency and maintain security of the data.

In addition, converting SONET ring problem to VRP provides flexibility in imposing additional constraint. A SONET ring that needs to send the signal within a certain time frame then the problem can be converted to VRP with time window Solomon[6]. Similarly if a SONET ring must not exceed a certain distance then the problem can be converted to Distance Constrained Vehicle Routing Problem (DCVRP) Laporte et al.[8].

Note: The depot in CVRP will be any node in SONET (single-route); also the pairs that they have a demand between each other will be treated as one customer.

4. Computations

This section applies the Clarke and Wright (C&W) classical heuristic, Simulating Annealing (SA) meta-heuristic and Branch and Cut (B&C) exact method to solve 4 CVRP benchmark problems. The problems can also consider as SONET network where every route should be seen as a ring. The details of the problems are illustrated in Table 1.

Figures 1, 2, 3 and 4 provide the results of applying C&W, SA and B&C to solve the benchmark problems.

Table 1. Benchmark Problems

Problem number	References	Number of customers
1	Eilon et al.[8]	13
2	Groetschel[9]	21
3	Eilon et al.[8]	31
4	Held and Karp[10]	48

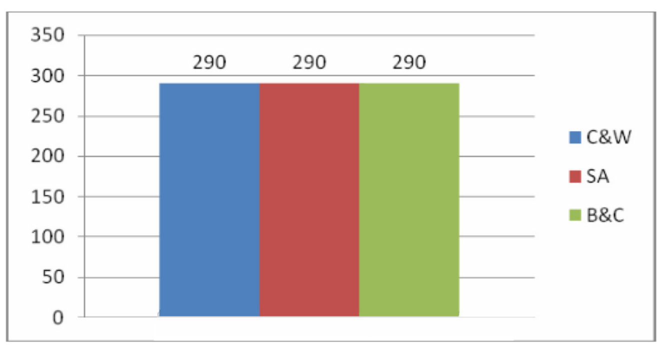

Figure 1. Problem 1, 13 customer.

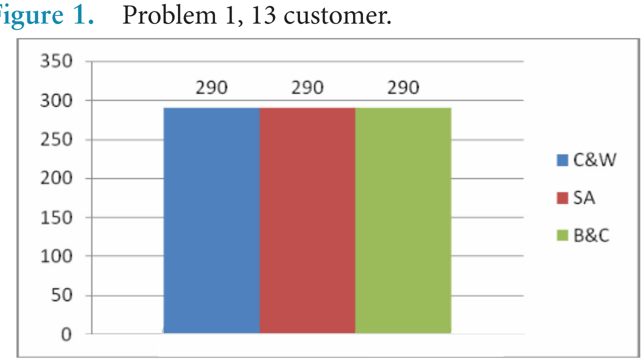

Figure 2. Problem 2, 21 customer.

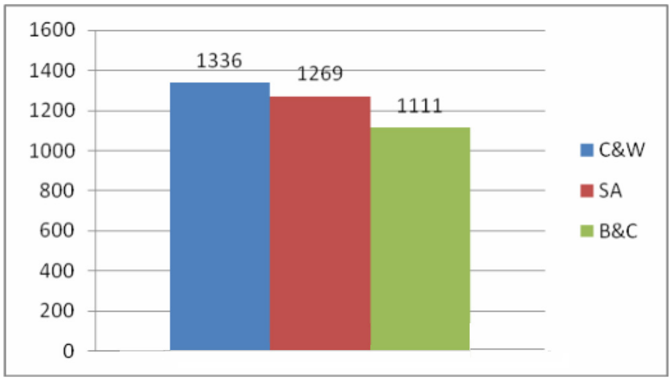

Figure 3. Problem 3, 31 customer.

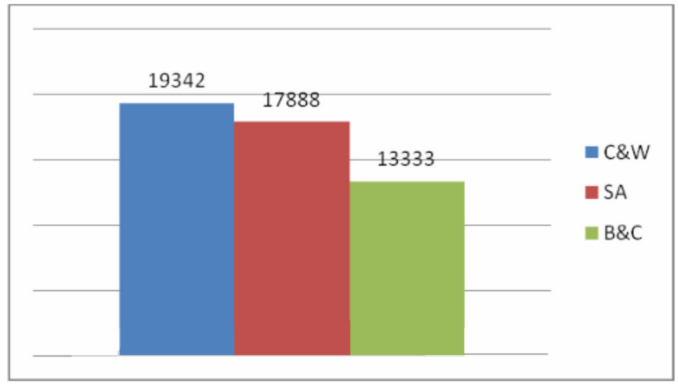

Figure 4. Problem 4, 48 customer.

5. Conclusion

Setting a SONET network is very expensive due to the high cost of the required equipments. Reducing the setting cost will maximize the profit rapidly. Using heuristics (both classical and meta-heuristics) may provide the optimal solution for small size problems (Figure 1). However, for bigger problems, only exact methods can give the optimal solution (Figures 2, 3 and 4). It's clear that the obtained results showed that converting SONET to CVRP provides the accuracy and the ability to optimize large scale problems. For future research, SONET other requirements (time and distance constraints) can be investigated with more details in order to be converted to the VRP with time window and distance constrained vehicle routing problem respectively.

6. Reference

1. Dantzig GB, Ramser JH. The Truck Dispatching Problem. Management Science. 1959; 6:80–91.
2. Achuthan NR, Caccetta L, Hill SP. A new sub tour elimination constraint for the vehicle routing problem. Eur J Oper Res. 1996; 91:573–86.
3. Bernardino E, Bernardino A, Sanchez-Perez J, Gomez-Pulido J, Vega-Rodriguez M. Solving large-scale SONET network design problems using bee-inspired algorithms.Optical Switching and Networking. 2012; 9:97–117.
4. Kim S, Kim Il-H, Mani V, Kim H. Ant Colony Optimization for SONET Ring Loading Problem. International Journal of Innovative Computing, Information and Control. 2008; 4:1617–26.
5. Karunanithi N, Carpenter T. Sonet ring sizing with genetic algorithms. Comput Oper Res. 1997; 24:581–91.
6. Solomon MM. Algorithms for the vehicle routing and scheduling problems with time window constraints. Oper Res. 1987; 35:254–65.
7. Laporte G, Desrochers M, Nobert Y. Two Exact Algorithms for the distance constrained vehicle routing problem. Networks. 1984; 14:161–72.
8. Eilon S, Watson-Gandy C, Christofides N. Distribution management, mathematical modeling and practical analysis. London: Griffin; 1971.
9. Groetschel M, Monma CL, Stoer M. Computational results with a cutting plane algorithm for designing communication networks with low connectivity constraints. Oper Res. 1992; 40:309–30.
10. Held M, Karp RM. The traveling salesman problem and minimum spanning trees. Oper Res. 1970; 18; 1138–62.